Universitatea "Alexandru Ioan Cuza" din Iaşi | **Admitere - studii de licenţă**

Facultatea de Informatică | **Sesiunea iulie 2017**

Barem de corectare la MATEMATICĂ

- Se acordă **10 puncte** din oficiu.
- Pentru orice soluţie corectă, chiar diferită de cea din barem, se acordă un punctaj corespunzător.

SUBIECTUL I **(30 de puncte)**

1.	$z=\frac{1}{\left(1+i\sqrt{2}\right)^2}+\frac{1}{\left(1-i\sqrt{2}\right)^2}=\frac{1}{-1+2\sqrt{2}i}+\frac{1}{-1-2\sqrt{2}i}$	**2p**
	$z=-\frac{2}{9}$	**2p**
	Concluzie: $Im(z)=0$.	**1p**
2.	Punctul de minim al parabolei este $V\left(-\frac{m+n}{2},-\frac{(m+n)^2+4m}{4}\right)$	**2p**
	Panta dreptei trebuie sa fie nulă $\Rightarrow m=0$	**1p**
	Dreapta trece prin $V\left(-\frac{n}{2},-\frac{n^2}{4}\right)\Rightarrow n^2+4n=0$	**1p**
	Concluzie: $n=0$ sau $n=-4$.	**1p**
3.	$x>1$	**1p**
	$x\pm 2\sqrt{x-1}=(\sqrt{x-1}\pm 1)^2\geqslant 0$	**1p**
	Ecuaţia devine $\lvert\sqrt{x-1}-1\rvert+\sqrt{x-1}+1+\log_2(x-1)=0$	**1p**
	Caz 1. $x\in(1,2)\Rightarrow 2+\log_2(x-1)=0\Rightarrow x=5/4$	**1p**
	Caz 2. $x\geqslant 2\Rightarrow 2\leqslant 2\sqrt{x-1}+\log_2(x-1)=0\Rightarrow$ ecuaţia nu are soluţii.	**1p**
4.	$P=\frac{f}{n}$	**1p**
	Numărul cazurilor posibile: $n=100$	**1p**
	Numărul cazurilor favorabile: $f=\text{card}\{1,2,4,8,16,32,64\}=7$ (justificare)	**2p**
	Concluzie: probabilitatea cerută este $P=\frac{7}{100}$.	**1p**
5.	$C(u,v)\in d\Leftrightarrow v=5-u$	**1p**
	$S_{\Delta ABC}=\frac{1}{2}\left\lvert det\begin{pmatrix}3&2&1\\1&0&1\\u&v&1\end{pmatrix}\right\rvert=2\lvert u-3\rvert=4$	**2p**
	Concluzie: $\lvert u-3\rvert=2\Rightarrow C(5,0)$ sau $C(1,4)$.	**2p**
6.	$\cos 2x=\cos x\Leftrightarrow 2\cos^2 x-\cos x-1=0\Leftrightarrow \cos x\in\{-1/2,1\}, x\in(0,\pi)\Rightarrow \cos x=-1/2$	**2p**
	$\cos 3x=4\cos^3 x-3\cos x=1$.	**3p**

SUBIECTUL al II-lea **(30 de puncte)**

1.a)	$A(x,y) \circ A(u,v) = A(xu - yv, xv + yu)$, $(xu-yv)^2 + (xv+yu)^2 = (x^2+y^2)(u^2+v^2) \neq 0$, $\forall A(x,y), A(u,v) \in G$	**1.5p**
	Asociativitatea operaţiei "$\circ$"	**1p**
	Comutativitatea operaţiei "$\circ$"	**0.5p**
	Elementul neutru al operaţiei "$\circ$" este $A(1,0)$	**1p**
	$\forall A(x,y) \in G,\ A(x,y)^{-1} = A\left(\dfrac{x}{x^2+y^2}, -\dfrac{y}{x^2+y^2}\right) \in G.$	**1p**
b)	φ este morfism: $\varphi((x+iy)\cdot(u+iv)) = \varphi(x+iy) \circ \varphi(u+iv), \forall (x+iy),(u+iv) \in \mathbb{C}^*$	**3p**
	φ este bijecţie	**2p**
c)	$A(0,-1) = \varphi(i)$	**2p**
	$A(0,-1)^{2017} = \varphi(i^{2017}) = \varphi(i)$	**2p**
	Concluzie: $A(0,-1)^{2017} = A(0,-1)$.	**1p**
2.a)	f se divide prin $X - \widehat{2} \Leftrightarrow f(\widehat{2}) = \widehat{0}$	**2p**
	$f(\widehat{2}) = p$	**2p**
	Concluzia: $p = \widehat{0}$.	**1p**
b)	$x_1^3 + x_2^3 + x_3^3 = -p(x_1^2 + x_2^2 + x_3^2)$	**1p**
	$x_1^2 + x_2^2 + x_3^2 = (x_1+x_2+x_3)^2 - 2(x_1x_2 + x_2x_3 + x_3x_1)$	**1p**
	$x_1 + x_2 + x_3 = -p$	**1p**
	$x_1x_2 + x_2x_3 + x_3x_1 = \widehat{0}$	**1p**
	Concluzia: $x_1^3 + x_2^3 + x_3^3 = -p^3$.	**1p**
c)	$f(\widehat{1}) = \widehat{0}$	**1p**
	$f = (X - \widehat{1})\cdot g, g = X^2 + \widehat{2}X + \widehat{2}$	**2p**
	g este ireductibil in $\mathbb{Z}_3$.	**2p**

SUBIECTUL al III-lea **(30 de puncte)**

1.a)	$f'(x)=2x(e^{x^2}-1), \forall x>0$	**1p**
	$f'(x)>0, \forall x>0$	**1p**
	f strict crescătoare pe $\mathbb{R}_+^*$	**1p**
	$\lim\limits_{x\searrow 0} f(x)=0$	**1p**
	$f(x)>0, \forall x\in\mathbb{R}_+^*$.	**1p**
b)	$g'(x)=\dfrac{(2x^2-1)e^{x^2}-x^2+1}{x^2}, \forall x>0$	**1p**
	$h(t)=(2t-1)e^t-t+1, \forall t\geqslant 0,\ h(0)=0$	**1p**
	$h'(t)=2te^t+e^t-1>0, \forall t>0$	**1p**
	h strict crescătoare pe $\mathbb{R}_+^*\Rightarrow h(t)>0, \forall t>0$	**1p**
	$g'(x)=h(x^2)/x^2>0, \forall x>0\Rightarrow g$ este crescătoare pe $\mathbb{R}_+^*$.	**1p**
c)	$\forall n\in\mathbb{N}, \exists c_n$ între x_{n-1} şi x_n astfel ca $x_{n+1}-x_n=f(x_n)-f(x_{n-1})=f'(c_n)(x_n-x_{n-1})$	**1p**
	$f'(c_n)>0\Rightarrow sgn(x_{n+1}-x_n)=sgn(x_n-x_{n-1}), \forall n\in\mathbb{N}^*$	**1p**
	$x_1=e^4-5>2=x_0\Rightarrow x_{n+1}>x_n, \forall n\in\mathbb{N}$	**1p**
	$(x_n)_{n\in\mathbb{N}}$ şir crescător $\Rightarrow \exists\lim\limits_{n\to\infty}x_n=l\in(2,\infty]$	**1p**
	$l<\infty\Rightarrow 1=\lim\limits_{n\to\infty}\dfrac{x_{n+1}}{x_n}=g(l)>g(2)>1$, absurd. Concluzie $l=\infty$.	**1p**
2.a)	$\displaystyle\int f(x)\,dx=\int 2x\sin^2 x\,dx=\int x(1-\cos 2x)\,dx=\frac{x^2}{2}-\int\frac{x}{2}(\sin 2x)'\,dx=\frac{x^2}{2}-\frac{x}{2}\sin 2x+$ $\displaystyle+\int\frac{\sin 2x}{2}\,dx=\frac{x^2}{2}-\frac{x}{2}\sin 2x-\frac{\cos 2x}{4}$	**3p**
	$\displaystyle\int f(x)+g(x)\,dx=x^2\Rightarrow\int g(x)\,dx=x^2-\int f(x)\,dx=\frac{x^2}{2}+\frac{x}{2}\sin 2x+\frac{\cos 2x}{4}$.	**2p**
b)	$I_n=\left(\dfrac{x^2}{2}-\dfrac{x}{2}\sin 2x-\dfrac{\cos 2x}{4}\right)\Bigg\vert_0^{1/n}=\dfrac{1}{2n^2}-\dfrac{1}{2n}\sin\dfrac{2}{n}-\dfrac{1}{4}\cos\dfrac{2}{n}+\dfrac{1}{4}, \forall n\in\mathbb{N}^*$	**2p**
	$J_n=\left(\dfrac{x^2}{2}+\dfrac{x}{2}\sin 2x+\dfrac{\cos 2x}{4}\right)\Bigg\vert_0^{1/n}=\dfrac{1}{2n^2}+\dfrac{1}{2n}\sin\dfrac{2}{n}+\dfrac{1}{4}\cos\dfrac{2}{n}-\dfrac{1}{4}, \forall n\in\mathbb{N}^*$	**2p**
	$I_n+J_n=\dfrac{1}{n^2}, \forall n\in\mathbb{N}^*$.	**1p**
c)	$n^2(J_n-I_n)=n\sin\dfrac{2}{n}+\dfrac{n^2}{2}\left(\cos\dfrac{2}{n}-1\right), \forall n\in\mathbb{N}^*$	**1p**
	$\lim\limits_{n\to\infty}n\sin\dfrac{2}{n}=2$	**1p**
	$\lim\limits_{n\to\infty}\dfrac{n^2}{2}\left(\cos\dfrac{2}{n}-1\right)=-\lim\limits_{n\to\infty}n^2\sin^2\dfrac{1}{n}=-1$	**2p**
	Concluzia: $\lim\limits_{n\to\infty}n^2(J_n-I_n)=1$.	**1p**