AN OPTIMAL DESIGN OF THE WIND TURBINE BLADE GEOMETRY ADAPTED TO A SPECIFIC SITE USING ALGERIAN WIND DATA

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Abstract

The aerodynamic modeling of the wind turbine blades constitutes one of the most important processes in the design of the turbine. The aim of this modeling is to calculate the aerodynamic loads, to determine the optimal parameters of the blades and estimate the wind extracted power. The design of the blade geometry must provide the optimal shape of the rotor blade capable to produce the maximum extracted power. This design must be done for a specific aerodynamic profile and a specific site, since this modeling must be based on statistical analysis of meteorological data of this given site.

In this work aerodynamic loads for small wind turbine blades are calculated as well as the total power extracted by the turbine. This design has a great impact on the turbine efficiency and consequently on its economical feasibility.

Keywords: Wind Energy, Aerodynamics, Numerical Methods, fluid mechanics.

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1. INTRODUCTION

Small wind turbine technology can be a meaningful contributor to long-term economic growth by assuring independence in energy supplies and providing benefits to local economy. Moreover wind is a clean non-polluting energy source and the electricity generated by this mean is becoming economically efficient compared to other sources. The aerodynamic modeling of the wind turbine blades constitutes one of the most important processes in the design of the turbine. The rotor blades are the most important part of this turbine because of their aerodynamic shape and profiles that play the main role in extracting the wind energy.

The aerodynamic modeling is used in order to estimate the aerodynamic loads and the wind extracted power. This modeling must be done for a given wind speed and a given rotor blades. The design of the blade geometry must provide the optimal shape of the rotor blade capable to produce the maximum extracted power. In order to determine the optimal shape of the blades, one must compute the optimal parameters of the blade geometry such as the chord length distribution, the thickness and the twist angle distribution along the blade span.

In the aerodynamic modeling two aerodynamic theories are used, the first one is the axial momentum theory and the second is the blade element theory [1]. In the first
theory, the flow is considered to be completely axial, while in the second theory the effect of wake rotation is included, assuming that the flow downstream rotates. The momentum theory that employs simply the mass and momentum conservation principles cannot provide alone the necessary information for the rotor design. However, the blade element theory that uses the angular momentum conservation principal, gives complementary information about the blade geometry such as airfoil shape and twist distribution. When both theories are combined the aerodynamic loads and the produced power can be obtained.

In order to compute the optimal parameters of the blade geometry that give the maximum power, an iterative algorithm is used until the maximum value of extracted power is reached. This design must be done for a specific aerodynamic profile and a specific site, since we must use the characteristic wind speed that gives the maximum available power in a given site. This characteristic wind speed is determined by statistical modeling of meteorological data. The design can be repeated for different sites and profiles.

The estimation of aerodynamic loads can be useful as well in strength calculation of the blades in order to predict the structural problems such as fatigue failure, which is the major cause of wind turbine breakdown.

This design has a great impact on the turbine efficiency and consequently on its economical feasibility.

2. AERODYNAMIC MODELING

In this aerodynamic modeling two aerodynamic theories are used.

2.1. AERODYNAMIC MODELING

In this simple one-dimensional model, airflow is assumed to be incompressible, completely axial and rotationally symmetric [2]. This model applies the principles of mass and momentum conservation on the annular control volumes surrounding the flow as shown in figure 1. In this figure, the symbol $A_i$ represents the section area at the station $i$, and the symbol $A$ is the section area at the rotor plane, while $V$ represents the wind speed.

The thrust force $T$ at the rotor disc can be found, by applying the conservation of linear momentum to the control volume in the axial direction:

$$ T = \dot{m}(V_0 - V_1) = \rho A V_0 (V_0 - V_1) $$  \hspace{1cm} (1)

where $\rho$ is the density of the air. Bernoulli’s equation can be applied to obtain the thrust as:

$$ (p - p')A = T = \frac{1}{2} \rho A (V_0^2 - V_1^2) $$  \hspace{1cm} (2)
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The power extracted from the wind by the rotor is:

\[ P = \frac{1}{2} \dot{m}(V_0^2 - V_1^2) = \frac{1}{2} \rho VA(V_0^2 - V_1^2) \]  

(3)

Introducing the axial interference factor, \( a \), which is defined as the fractional decrease in wind velocity between the free stream and the rotor plane:

\[ V = (1 - a)V_0 \]  

(4)

The thrust expression of equation (2) becomes:

\[ T = \frac{1}{2} \rho AV_0^2 4a(1 - a) \]  

(5)

The power extracted by the rotor is:

\[ P = \frac{1}{2} \rho AV_0^2 4a(1 - a)^2 \]  

(6)

2.2. THE BLADE ELEMENT THEORY

This analysis uses the angular momentum conservation principle, taking into account the blade geometry characteristics, in order to determine the forces and the torque exerted on a wind turbine. This method is known as blade element theory [3]. The control volume used in the previous one-dimensional model can be divided into several annular stream tube control volumes, which split the blade into a number of
distinct elements, each of length \( dr \) (figure 2). In this figure, the symbol \( w_i \) represents the wake angular velocity at the station \( i \), and \( w \) is the wake angular velocity at the rotor plane.

In this theory it is assumed that there is no interference between these blade elements and these elements behave as airfoils.

The differential rotor thrust, \( dT \), at a given span location on the rotor (at a specified radius \( r \)) can be derived from the previous theory using equation (5):

\[
dT = 4a(1 - a)\rho V_0^2 \pi rdr
\]  
(7)

In the previous model, it was assumed that airflow doesn’t rotate. However, the conservation of angular momentum implies the rotation of the wake, if the rotor is to extract useful torque. Moreover, the flow behind the rotor will rotate in the opposite direction [3], as shown in figure 3.

The effect of wake rotation will be now included. In describing this effect, the assumption is made that upstream of the rotor, the flow is entirely axial and that the flow downstream rotates with an angular velocity \( \omega \).

The conservation of angular momentum can be applied to obtain the differential torque at the rotor disc, \( dQ \), resulting in:

\[
dQ = 2\pi \rho V_0 \omega^3 dr
\]  
(8)

The total torque is:

\[
Q = 2\pi \rho \int_0^R V\omega r^3 dr
\]  
(9)

Fig. 2.: Annular stream tube control volumes
The differential extracted power is given by the expression:

$$dP = 2\pi \rho V \omega r^3 dr$$  \hspace{1cm} (10)

The total extracted power is:

$$P = 2\pi \rho \Omega \int_0^R V \omega r^3 dr$$  \hspace{1cm} (11)

In order to calculate $P$ and $Q$, the wake angular velocity $\omega$ has to be known. Introducing, for this purpose, the tangential interference factor $a'$ defined as:

$$\omega = a' \Omega$$  \hspace{1cm} (12)

The differential lift and drag forces are:

$$dL = C_L dq$$  \hspace{1cm} (13)
$$dD = C_D dq$$  \hspace{1cm} (14)

with:

$$dq = \frac{1}{2} \rho W^2 dA = \frac{1}{2} \rho W^2 c dr$$  \hspace{1cm} (15)

where $C_L$ and $C_D$ are lift and drag coefficients.

The components of the resulting force are (see figure 4.):

$$dF_x = C_s dq$$  \hspace{1cm} (16)
$$dF_y = C_v dq$$  \hspace{1cm} (17)

where:

$$C_s = C_L \sin \phi - C_D \cos \phi$$  \hspace{1cm} (18)
The following relation can be derived from figure 4:

\[\tan \phi = \frac{(1 - a)V_0}{(1 + a')\Omega r}\]  

(20)

where:

\[\alpha = \phi - \beta\]  

(21)

The differential thrust and torque can now be derived as follows:

\[dT = BC_y dq = BC_y \frac{1}{2} \rho W^2 c dr\]  

(22)

\[dQ = BC_x dqr = BC_x \frac{1}{2} \rho W^2 c r dr\]  

(23)

where:

\(\Omega\): is the rotation speed of the rotor
\(W\): is the wind relative speed
\(c\): is the chord length

Equating the thrust in equations (7) and (22) as well as the torque in equations (8) and (23), will yield to the expressions of the both interference factors:

\[a = \frac{1}{\frac{4 \sin^2 \phi}{\sigma c_y} + 1}\]  

(24)
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\[ a' = \frac{1}{\frac{4 \sin \phi \cos \phi}{\sigma C_s} - 1} \]  

(25)

The local solidity \( \sigma \) is defined by the following formula:

\[ \sigma = \frac{cB}{2\pi r} \]

(26)

where: \( B \) is the number of blades.

In order to carry out this calculation:

- The wind speed must be determined from the graph of the power probability density (figure 9).
- This calculation procedure is repeated iteratively till the values of the interference factors are corrected.
- In order to get the optimal blade geometry that can extract the maximum power, one must use the optimal incidence angle \( \alpha_{opt} \) that gives \( \left[ \frac{C_L}{C_D} \right]_{Max} \)

The distributions of aerodynamic loads (at different blade stations) due to a wind speed of 15 m/s for a blade having a NACA 63-421 profile, are given by table 1.

<table>
<thead>
<tr>
<th>Station ((r/R))</th>
<th>Axial force (N)</th>
<th>Tangential force (N)</th>
<th>Torque (N.m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.16</td>
<td>86.02</td>
<td>221.24</td>
<td>206.30</td>
</tr>
<tr>
<td>0.25</td>
<td>81.92</td>
<td>351.16</td>
<td>305.56</td>
</tr>
<tr>
<td>0.34</td>
<td>73.37</td>
<td>466.19</td>
<td>372.82</td>
</tr>
<tr>
<td>0.43</td>
<td>57.87</td>
<td>586.57</td>
<td>467.49</td>
</tr>
<tr>
<td>0.51</td>
<td>35.67</td>
<td>764.83</td>
<td>724.62</td>
</tr>
<tr>
<td>0.60</td>
<td>39.33</td>
<td>908.37</td>
<td>1120.22</td>
</tr>
<tr>
<td>0.69</td>
<td>35.60</td>
<td>998.27</td>
<td>1686.21</td>
</tr>
<tr>
<td>0.78</td>
<td>221.50</td>
<td>1012.80</td>
<td>2591.9</td>
</tr>
<tr>
<td>0.87</td>
<td>211.31</td>
<td>1109.45</td>
<td>2746.15</td>
</tr>
<tr>
<td>0.96</td>
<td>169.79</td>
<td>1181.84</td>
<td>2434.31</td>
</tr>
<tr>
<td>1.00</td>
<td>140.04</td>
<td>1206.90</td>
<td>2100.65</td>
</tr>
</tbody>
</table>

The local (differential) loads obtained are integrated numerically, over the length of the blade, to determine the overall aerodynamic loads as well as the total output power. Their values are given as follows:

Total axial force on one blade = 553.24 N
Total tangential force on one blade = 4729.56 N
Total axial force on the rotor= 1659.72N
Total torque = 8218.81 N.m
3. OPTIMIZATION OF THE BLADE GEOMETRY

In this part the blade optimal form (chord length and twist angle) is determined. It has been established, that the extracted power of the turbine is given by the following expression [2]:

\[ P = \frac{1}{2} \rho \pi R^2 V_0^3 \cdot \frac{4a^2}{R^4} \int_0^R (1 - a') a' r^3 \, dr \]  

(27)

The power coefficient is defined as follows:

\[ C_p = \frac{P}{\frac{1}{2} \rho V_0^3 A} = \frac{Extracted\, power}{Available\, Power} \]  

(28)

The extracted Power reaches its maximum value when \( C_p \) is maximum. It has been demonstrated that \( C_p \) is maximum when:

\[ a' = \frac{(1 - 3a)}{(4a - 1)} \]  

(29)

Thorough calculation will yield that the optimal chord length is given by the expression:

\[ C = \frac{8\pi r}{BC_L} (1 - \cos \theta) \]  

(30)

and the optimal twist is expressed as follows:

\[ \beta = \phi - \alpha_{opt} \]  

(31)

where: \( \theta = \frac{2}{3} \tan^{-1} \left( \frac{V}{\Omega r} \right) \) and the optimal incidence angle \( \alpha_{opt} \) is obtained when the ratio of the lift coefficient to the drag coefficient is maximum, that means: \( \alpha = \alpha_{opt} \Rightarrow \left( \frac{c_l}{c_d} \right)_{max} \).

The optimal distribution of the twist angle and the chord length are given by figure 5 and 6.

4. STATISTICAL ANALYSIS OF WIND DATA

In this part a statistical analysis is carried out in order to estimate wind characteristics in different regions of Algeria and accordingly determine the most efficient sites. The statistical distribution of wind speed (the frequency of occurrence of each speed) is also valuable in making an optimal design of wind turbine such as in fatigue failure prediction as well as in the adaptation of a machine to a site [4].

4.1. EXPERIMENTAL WIND DATA

STATISTICAL DISTRIBUTION

The estimation of the wind resources presents a particular difficulty because of the variability of wind speed characteristics which varies with the season and the hour of
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The statistical distribution of wind speed characteristics varies also from one place to another, since it depends on the local climatic conditions and the landscape of the site. Wind characteristics such as: average speed, speed frequency and directions facilitate...
the estimation of the total energy extracted by wind turbine. These parameters have a direct influence on the operation of turbine (starting, stopping, orientation etc.)
This information is also required to optimize the design of the wind turbine in order to maximize the extracted energy and therefore to minimize the electricity production cost [6].
In order to determine the wind properties of a site, wind data must be available over a long period of time (one to ten years).

![Fig. 7.: Probability Density of wind speed (Town of Adrar)](image)

### 4.2. ESTIMATION OF THE AVERAGE AVAILABLE POWER (POWER DISTRIBUTION)

The available wind power per unit area varies proportionally with the cube of the wind speed, as follows:

\[ P = \frac{1}{2} \rho V^3 \]  

(32)

where \( \rho \) is the density of the air, \( P \) is the available power of the wind per unit of area. By multiplying the available power (at each wind speed) by the probability of occurrence of this speed, one obtains the distribution of the wind power at various speeds
of wind. This last distribution is also called the power density [7]. It is important

![Fig. 8.: probability Density of the average power (Town of Setif)](image1)

![Fig. 9.: Probability Density of the average power (Town of Tindouf)](image2)

... to note that at speeds higher than the mean velocity, in a given site, where the major part of power is produced.

The optimal design of the wind rotor, for a given site, must be based on the speed of
wind that gives the maximum available energy. For the town of Setif (Figure 8) this speed is 7 m/s.

5. CONCLUSION

In this work the blade element theory was used to calculate aerodynamic loads for small wind turbine blades. This method can also estimate the power extracted by the turbine. This design must be done for a specific aerodynamic profile and a specific site, since we must use the characteristic wind speed that gives the maximum available power in a given site. The value of this characteristic wind speed is determined from the wind data statistical analysis of this site. The design can be repeated for different sites and profiles until we determine the right blade profile that suit each site.

According to results of the wind statistical analysis Tindouf has the highest average power (figure 9). This average power is the most determinant parameter to use in the selection of the wind site.

Actually the criterion of maximum annual energy production used to optimize blade geometry is not sufficient. Whereas the optimization of wind turbine must be based on minimum cost of energy which requires a multidisciplinary method that includes aerodynamic and structural models for blades along with a cost model for the whole turbine [8]. This work can be a part of a global optimization study aiming to minimize cost and structural problems of wind turbine while maximizing its energetic performance.

References