EXAMPLES OF $L$-PARTITIONS OF $L$-FUZZY SETS

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Abstract If $X$ is a set and $L$ a lattice, then a function $\varphi : X \to L$ is called a fuzzy $L$-subset of $X$. In [1], by generalizing similar results about partitions of fuzzy subsets, $L$-partitions of $L$-fuzzy subsets are presented. In this paper we give examples of $L$-partitions of $L$-fuzzy subsets.

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1. INTRODUCTION

Because the conventional computer logic was not capable to treat the data representing vague values of some properties and parameters, L. A. Zadeh introduced the fuzzy sets theory in his work “Fuzzy sets” published in 1965. While the classic sets can be represented by their characteristic function (membership function) which takes the values 0 or 1, the elements in fuzzy sets have different degrees of membership from 0 to 1.

Fuzzy sets and fuzzy logic opened ways of research in many areas such as: artificial intelligence, multicriteria decision making, design of controlled systems, databases query, image processing, data analysis.

Fuzzy logic is very useful for Case Based Reasoning Systems because the analogical reasoning can operate with linguistic expressions and the fuzzy logic is created to operate with linguistic expressions.

Similarity measurement and the cases classifications are based on fuzzy sets partitions which allowed to develop many fuzzy models.

In [1], the author generalized fuzzy partitions to fuzzy $L$-partitions and presented some results regarding these notions. These results are presented in Section 2 without proofs. In Section 3 some examples of fuzzy $L$-partitions, which is the aim of this paper, are described.
2. \textbf{L- FUZZY SETS AND L- FUZZY PARTITIONS}

In this section we introduce some notions and results from [1] on \(L\)-fuzzy sets and \(L\)-fuzzy partitions.

\textbf{Definition 2.1.} Let \(X\) be a set and let \(L\) be a lattice. A function \(\varphi : X \to L\) is called fuzzy \(L\)-subset of the set \(X\). Consider \((L, \land, \lor, 0, 1)\) a complete distributive lattice and \((-)' : (L', \land, \lor, 0, 1) \to (L, \lor, \land, 1, 0)\) an involutive anti-isomorphism such that \((x')' = x\), \((x \land y)' = x' \lor y'\), \((x \lor y)' = x' \land y'\) for any \(x, y \in L\). Suppose that there exists an element \(x_0\) in \(L\) such that \((x_0')' = x_0\) and for any \(x \in L\), \(x \leq x_0\) or \(x \geq x_0\). This kind of lattice exists. Indeed, for example we may consider the lattice \(L = ([0, \frac{1}{2}], \land, \lor, 0, \frac{1}{2})\) and \(L' = ([\frac{1}{2}], \land, \lor, 1, \frac{1}{2})\) and \(x' = 1 - x\) for any \(x \in L\). We can see that \(x_0 = \frac{1}{2}\) satisfies the required conditions and \((-)'\) is an involutive antiisomorphism satisfying DeMorgan's laws.

\textbf{Proposition 2.1.} If the lattice \(L\) and the element \(x_0\) are above defined, then the following properties are satisfied:
1) \(x \leq x'\) iff \(x \leq x_0\);
2) \(x \land x' \leq x_0 \leq x \lor x'\) for any \(x\) in \(L\).

\textbf{Remark 2.1.} \((\sup_k x_k)' = \inf_k x'_k\)

\textbf{Definition 2.2.} Let \(X\) be a set and let \(\varphi\) be an \(L\)-fuzzy subset of \(X\). A family of \(L\)-fuzzy subsets \((\nu_n)\) that satisfies the following conditions:
1) \(\nu_i \leq \nu_j', \forall i \neq j\)
2) \(\varphi \land (\sup_n \nu_n)' \leq (\varphi \land (\sup_n \nu_n))'\)
3) \(\sup_n \nu_n \leq \varphi\)

is called a partition of \(\varphi\).

In case \(\varphi = 1_L\) the family \((\nu_n)\) is called a complete partition.

\textbf{Theorem 2.1.} The family \((\nu_n)\) is a complete partition iff it satisfies the following conditions:
1) \(\nu_i \leq \nu_j', \forall i \neq j\),
2) \(\sup_n \nu_n \geq (\sup_n \nu_n)'\).

\textbf{Theorem 2.2.} Assuming \(\sup_n \varphi_n \geq \sup_n (\varphi_n)'\), the sequence \((\nu_n)\) defined below is a complete partition

\[
\nu_n = \begin{cases} 
\varphi_1, & \text{if } n = 1 \\
\varphi_n \land (\sup \nu_k)', & \text{if } n > 1 
\end{cases}
\]

\textbf{Theorem 2.3.} If the family \((\nu_n)\) is a complete partition, then the family \((\varphi \land \nu_n)\) is a partition of \(\varphi\) for any \(L\)-fuzzy subset \(\varphi\).
3. EXAMPLES OF FUZZY L- PARTITIONS

Consider the set \( X = [0, 1] \), the lattice \( L = ([0, 1], \land, \lor, 0, 1) \), \( x' = 1 - x \) and a family of three \( L \)-fuzzy subsets of \( X \) \( \nu_1, \nu_2, \nu_3 : X \to L \) defined below

\[
\nu_1 = \begin{cases} 
1, & \text{if } x \in [0, \frac{1}{2}] \\
-6 \cdot x + \frac{5}{2}, & \text{if } x \in \left[\frac{1}{2}, \frac{5}{12}\right] \\
0, & \text{if } x \in \left(\frac{5}{12}, 1\right]
\end{cases}
\]

\[
\nu_2 = \begin{cases} 
0, & \text{if } x \in [0, \frac{1}{2}] \\
6 \cdot x - \frac{3}{2}, & \text{if } x \in \left[\frac{1}{2}, \frac{5}{12}\right] \\
-6 \cdot x + \frac{7}{2}, & \text{if } x \in \left[\frac{5}{12}, \frac{7}{12}\right] \\
0, & \text{if } x \in \left(\frac{7}{12}, 1\right]
\end{cases}
\]

\[
\nu_3 = \begin{cases} 
0, & \text{if } x \in [0, \frac{5}{12}] \\
6 \cdot x - \frac{5}{2}, & \text{if } x \in \left[\frac{5}{12}, \frac{7}{12}\right] \\
1, & \text{if } x \in \left(\frac{7}{12}, 1\right]
\end{cases}
\]

It follows that

\[
\nu_1'(x) = \begin{cases} 
0, & \text{if } x \in [0, \frac{1}{2}] \\
-6 \cdot x + \frac{7}{2}, & \text{if } x \in \left[\frac{1}{2}, \frac{5}{12}\right] \\
0, & \text{if } x \in \left(\frac{5}{12}, 1\right]
\end{cases}
\]

\[
\nu_2'(x) = \begin{cases} 
1, & \text{if } x \in [0, \frac{1}{2}] \\
-6 \cdot x + \frac{5}{2}, & \text{if } x \in \left[\frac{1}{2}, \frac{5}{12}\right] \\
6 \cdot x - \frac{7}{2}, & \text{if } x \in \left[\frac{5}{12}, \frac{7}{12}\right] \\
0, & \text{if } x \in \left(\frac{7}{12}, 1\right]
\end{cases}
\]

\[
\nu_3'(x) = \begin{cases} 
1, & \text{if } x \in [0, \frac{5}{12}] \\
-6 \cdot x + \frac{7}{2}, & \text{if } x \in \left[\frac{5}{12}, \frac{7}{12}\right] \\
1, & \text{if } x \in \left(\frac{7}{12}, 1\right]
\end{cases}
\]

With the graphs of these functions and by a simple computation we can easily check that the following conditions are satisfied:
1) \( \nu_i \leq \nu_j' \), \( i, j \in \{1, 2, 3\}, i \neq j \),
2) \( \sup\{\nu_1, \nu_2, \nu_3\} \geq \sup(\{\nu_1, \nu_2, \nu_3\})' \).

Then, according to Theorem 2.1, it follows that the family \( \{\nu_1, \nu_2, \nu_3\} \) is a complete fuzzy \( L \)-partition.

With \( \nu_1, \nu_2, \nu_3 \) above defined let us consider the following fuzzy \( L \)-subsets
\( \mu_1, \mu_2, \mu_3 \)
\( \mu_1 = \nu_1 \)
\( \mu_2 = \nu_2 \land \mu_1' = \nu_2 \land \nu_1' \)
\( \mu_3 = \nu_3 \land (\sup\{\nu_1, \nu_2\})' = \nu_3 \land \inf\{\nu_1', \nu_2'\} \)

According to Theorem 2.2, the family \( \{\mu_1, \mu_2, \mu_3\} \) is a complete fuzzy \( L \)-partition.
If $L$ is an $L$-fuzzy subset, then the families $\{\varphi \land \nu_1, \varphi \land \nu_2, \varphi \land \nu_3\}$ and $\{\varphi \land \mu_1, \varphi \land \mu_2, \varphi \land \mu_3\}$ are fuzzy $L$-partitions of $\varphi$ according to Theorem 2.3.

References