APPROXIMATE STABILITY SURFACES IN A CONVECTION PROBLEM FOR A MICROPOLAR FLUID. NUMERICAL RESULTS FOR THE HYDRODYNAMIC CASE

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Abstract In [2] the stability problem of thermal convection in a heat conducting micropolar fluid layer between rigid boundaries was treated theoretically using direct and variational methods. In this paper, for the same problem, we investigate the approximate numerical values of the Rayleigh number in the hydrodynamic case for different values of the physical parameters.

Keywords: stability, micropolar fluid, thermal convection, Rayleigh number

2000 MSC: 65L15,34K20,34K28 **Introduction**. We present a numerical study of the onset of thermal convection in a heat conducting micropolar fluid layer between two rigid boundaries. This particular problem of stability was solved theoretically in [2] using two direct and variational methods. We recall that these methods are based on the fact that the space $L^2(a,b)$ is a separable Hilbert space [5].

As it was stated in [2], assuming that the exchange of stability principle holds [9], the linear stability against normal mode perturbations is governed by the two-point problem

$$\begin{cases} (1+R)\Big[(D^2-a^2)^2-QD^2\Big]W+R(D^2-a^2)Z-Ra\cdot a^2\Theta=0, \\ \Big[A(D^2-a^2)-2R\Big]Z-R(D^2-a^2)W=0, \\ (D^2-a^2)\Theta+W-\overline{\delta}Z=0, \end{cases} \tag{1}$$

$$W=DW=Z=\Theta=0 \text{ at } z=\pm 0.5. \tag{2}$$

The micropolar parameters are $R=\dfrac{k}{\mu}$, $A=\dfrac{\gamma}{\mu d^2}$, $\overline{\delta}=\dfrac{\delta}{\varrho_0 c_v d^2}$, Q is the intensity of the magnetic field, Ra stands for the Rayleigh number and $a>0$ is the wave number. The numbers μ, k, α, β, γ and δ are material constants. The functions W, Θ, $Z:[-0.5,0.5]\to\mathbb{R}$ characterize the amplitude of the perturbation of the vertical component of the velocity, temperature and the vertical component of the spin vorticity, respectively.

One of the methods used in [2] is the Budiansky-DiPrima method, based on the expansion of the unknown functions upon total sets of functions which do not satisfy all boundary conditions of the problem.

Each of the unknown functions from (1) can be written as a sum of an odd function and an even function. In this way the problem splits into a problem with even functions and one problem with odd functions. Let us consider here the even part of the problem. In this case, the unknown functions W, Θ, Z are even functions, so we expand them upon the total set $\{E_{2n-1}\}_{n\in\mathbb{N}}$, $E_{2n-1}(z) = \sqrt{2}\cos(2n-1)\pi z,\ n \in \mathbb{N}$, namely

$$W = \sum_{n=1}^{\infty} W_{2n-1}E_{2n-1}(z), Z = \sum_{n=1}^{\infty} Z_{2n-1}E_{2n-1}(z), \Theta = \sum_{n=1}^{\infty} \Theta_{2n-1}E_{2n-1}(z). \quad (3)$$

The condition $DW = 0$ at $z = \pm 0.5$ is not satisfied such that it introduces a constraint for the problem (1)-(2).

The expressions of the derivatives occuring in (1) are obtained by the backward integration technique [5]. Substitute these expressions in (1), impose the condition that the obtained equations be orthogonal to E_{2m-1}, $m = 1, 2, ...$ to get the system

$$\begin{cases} (1+R)[A_n^2 + Q(2n-1)^2\pi^2]W_{2n-1} - RA_nZ_{2n-1} - Ra\cdot a^2\Theta_{2n-1} = \\ = 2\sqrt{2}(-1)^n(1+R)(2n-1)\pi\alpha \\ \\ RA_nW_{2n-1} - (AA_n + 2R)Z_{2n-1} = 0, W_{2n-1} - \overline{\delta}Z_{2n-1} - A_n\Theta_{2n-1} = 0, \end{cases} \quad (4)$$

with the constraint

$$\sum_{n=1}^{\infty}(-1)^n\sqrt{2}(2n-1)\pi W_{2n-1} = 0, \quad (5)$$

where $A_n = (2n-1)^2\pi^2 + a^2$.

The secular equation is obtained by solving the system (4) and replacing the obtained expression for W_{2n-1} in (5). **Numerical results**. The numerical study is done only for the hydrodynamic case, i.e. $Q = 0$. In this two cases ($Q = 0, \overline{\delta} = 0$), ($Q = 0, \overline{\delta} \neq 0$) we perform numerical computations and we obtain approximate values of the Rayleigh number for various values of the physical parameters. Also the neutral cureves and surfaces are drawn in various parameter spaces.

Case $Q = 0,\ \overline{\delta} = 0$. In this case the secular equation has the form

$$\sum_{n=1}^{\infty} \frac{(2n-1)^2A_nD_n}{A_n^3\Big[D_n(1+R) - R^2\Big] - Ra\cdot a^2D_n} = 0. \quad (6)$$

Since this series is convergent, performing numerical evaluations for (6) we obtained values for the Rayleigh number. For different values of the micropolar parameters R, A and of the wave number a some of them are presented in Table 1. The results show that as R increases, the value of the Rayleigh number increases rapidly. Also, the growth of the wave number a implies a steep growth of the Rayleigh number. When the parameter A modify, the changes in the values of the Rayleigh number are not significant. From the numerical results it seems that the biggest influence on the values of the Rayleigh number has the parameter R.

A	R	a	Ra
0.001	0.001	3.117	1850.624086
0.001	0.5	3.117	2302.200180
0.001	0.5	5.00	3213.376953
0.001	1.00	5.00	3860.174964
0.002	1.00	5.00	3911.264998
0.001	2.00	6.70	9015.560744
0.001	2.00	14.00	93743.02414
0.001	2.00	9.50	24618.15747
0.001	4.00	9.50	36480.43048
0.001	6.00	9.50	48513.80933
0.001	8.00	9.50	60537.15202

Table1 : Approximate Rayleigh number for $Q = 0, \overline{\delta} = 0$.

When the micropolar parameters R and A are very small (close to zero), we found the classical case treated by Chandrasekar [1] (fig.1). The neutral curve is also similar to the one in [1].

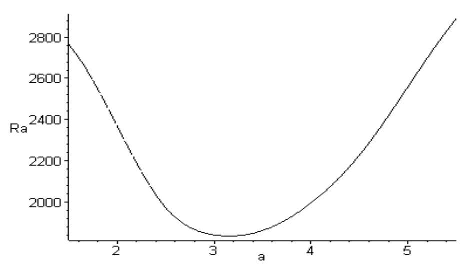


Fig.1 : The approximate Rayleigh number at which instability sets in for $Q = 0, \overline{\delta} = 0, A = 0.001$.

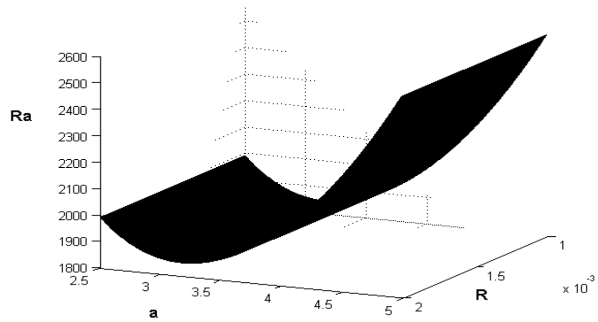


Fig.2 : The approximate neutral surface in the space (R, a, Ra) for small values of the parameters R and A.

Case $Q = 0$, $\overline{\delta} \neq 0$. If the intensity of the magnetic field is zero, but the micropolar parameter $\overline{\delta} \neq 0$, the secular equation has the form

$$\sum_{n=1}^{\infty} \frac{(2n-1)^2 A_n D_n}{A_n^3 \left[D_n(1+R) - R^2 \right] + Ra \cdot a^2 (\delta R A_n - D_n)} = 0. \tag{7}$$

This series is also convergent.

To verify the obtained numerical values, we selected, at the beginning, the same values for the micropolar parameters and for the wave number as in [11]. The differences between the two numerical evaluations are found to be small. In carrying out the numerical computations, we have taken more values for the parameters than in [11], so that we noticed the different influences that the parameters have on the Rayleigh number.

A	R	$\overline{\delta}$	a	Ra
0.001	2	0.1	6.90	−5204.902
0.001	4	0.1	6.80	−7723.6289
0.001	6	0.1	6.85	−10259.840
0.001	8	0.1	6.85	−12785.852
0.001	2	0.05	9.00	−16641.5305
0.001	2	0.02	14.00	−97263.438
0.005	2	0.1	6.75	−5863.3438
0.01	2	0.1	6.70	−6723.4570
0.05	2	0.1	6.70	−17143.020

a) Critical values of $R_a$obtained in [8].

A	R	$\overline{\delta}$	a	Ra
0.001	2	0.1	6.90	−5218.031767
0.001	4	0.1	6.80	−7778.615586
0.001	6	0.1	6.85	−10271.62894
0.001	8	0.1	6.85	−12836.01081
0.001	2	0.05	9.00	−16674.77845
0.001	2	0.02	14.00	−97384.66469
0.005	2	0.1	6.75	−5909.801271
0.01	2	0.1	6.70	−6763.317816
0.05	2	0.1	6.70	−17344.22361
0.01	2	0.05	6.70	−44526.37592
0.05	2	0.05	6.70	59310.64812
0.05	4	0.05	6.70	−870390.2727

b) Critical values of $R_a$obtained by us.

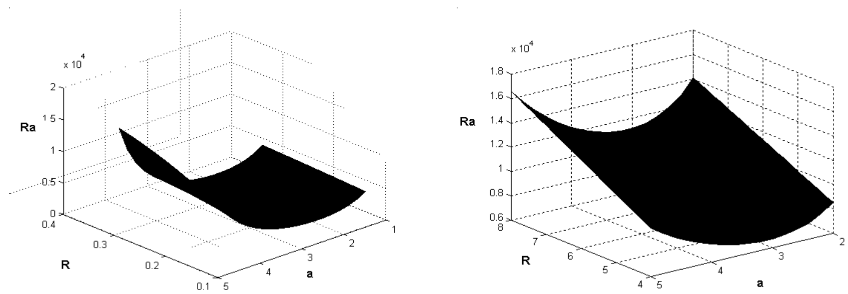
Table 2: Critical values of the Rayleigh number in the case $Q = 0$, $\overline{\delta} \neq 0$.

a) $A = 0.001,\ \overline{\delta} = 0.05$ *b*) $A = 0.001,\ \overline{\delta} = 0.01$

Fig.3 : The approximate neutral curve in the parameter space (a, Ra) in the case $Q = 0,\ \overline{\delta} \neq 0$.

Conclusions. In this paper we approximates the values for the Rayleigh number on the neutral surface. The evaluations showed that when the micropolar parameter $\overline{\delta}$ is not null, the viscosity parameter has a stabilizing influence on the flow. When A and $\overline{\delta}$ increases, large values of the wave number seems to have a stabilizing effect on the fluid.

When $\overline{\delta} = 0$, we can also treat the problem using the variational Budiansky-DiPrima method [2]. In this case, the operator is selfadjoint, so the evaluations are simplified since the number of derivatives is half of the derivatives occuring when we use the direct Budiansky-DiPrima method. The secular equation obtained in this case is the same. The direct Budiansky-DiPrima method was chosen to perform the numerical evaluations for the Rayleigh number so that we avoid very difficult numerical evaluations.



a) $A = 0.001,\ \overline{\delta} = 0.05$ $\qquad$ $A = 0.001,\ \overline{\delta} = 0.001$

Fig.4 : The approximate neutral surface in the parameters space(a, R, Ra) in the case $Q = 0,\ \overline{\delta} \neq 0$.

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