Stability of equilibria in an infinite dimensional network of theta neurons

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Motivation: We consider an infinite network of identical theta neurons, all-to-all coupled by instantaneous synapses, modeled by a system of delay differential equations. The state of neuron k at time t is $\theta_k(t) \in [0, 2\pi]$ and the dynamics of the network is described by the following system of differential equations:

$$\frac{d\theta_k}{dt} = 1 - \cos\theta_k + (1 + \cos\theta_k)(\eta + \kappa I), \quad \text{for } k = \overline{1, N} \tag{1}$$

where κ denotes the overall coupling strength (positive or negative), η is the input current to all neurons when uncoupled, and for I, two separate cases are considered:

- in the non-delayed case:

$$I = \frac{1}{N} \sum_{j=1}^{N} (1 - \cos\theta_j)^2 \tag{2}$$

- in the delayed case:

$$\frac{dI}{dt} = \frac{1}{\tau} \left(\frac{3}{2} - 2\rho\cos\Phi + \frac{\rho^2 \cos 2\Phi}{2} - I \right) \tag{3}$$

Methodology of Research: C. R. Laing. The dynamics of networks of identical theta neurons.Journal of Mathematical Neuroscience, 8:4, 2018
Conclusions: Using the Watanabe-Strogatz ansatz transformation which reduces the dimension of the infinite network, leading to a three-dimensional system of differential equations, we determine the number of equilibria with respect to κ and η, the characteristic parameters of the system.
Acknowledgements: Furthermore, we discuss the stability properties of each equilibrium and the possible bifurcations that may take place in the system. Numerical results are also presented to illustrate complex dynamic behavior in the neural system with or without time delays. We conclude that the number of equilibria and their stability properties is not affected by the introduction of the time delay as in (3), the only notable effect being that a third eigenvalue is gained. As a direction for future work, the case of a discrete time delay will be considered in system (1), i.e. $I = I(t - \tau)$ according to equation (2).

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