# The dynamics around the Earth and other extended bodies

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## Abstract

This presentation describes some tools and techniques that are used to investigate the dynamics of space debris and the dynamics around the Earth or some irregularly shaped bodies, such as asteroids and comets. These tools include the dimension reduction of a Hamiltonian problem, Fast Lyapunov Indicators and Polyhedron dynamics.

Dimension reduction can be used to study the phase space of some general models. For example, in the case of resonances of any kind, it is often possible to construct a toy model including some particular resonant terms and to reduce the number of degree of freedom of the original problem. Some examples will be shown where it is possible to reduce a general problem to a simple (and integrable) 1-d.o.f. model. Some results describing the so-called Solar radiation pressure resonances in the Earth's environment will be provided. Moreover, there is shown how the framework would change for asteroids' (or comets) environments.

Fast Lyapunov Indicators (FLIs) are a powerful numerical tool to study the presence of chaos in a given dynamical system. We will recall some definitions and applications to resonant dynamics.

Finally, the talk will discuss polyhedron dynamics, a topic closely connected with the FLAIR Project Working Group within the Stardust-r MCSA ITN. This theory describes, using simple formulas, the gravitational potential of a three dimensional polyhedron, which is generally used to approximate an irregularly shaped body such as an asteroid. The talk will describe some known results from this area of study, will address possible applications to asteroid dynamics and will discuss how this theory is used in the FLAIR PWG. No original results using this method are expected to be shown.

**Keywords**: Celestial Mechanics; Hamiltonian Mechanics; Resonances; Polyhedron Dynamics
**Domain**: mathematics
**Section**: Elaboration of the doctoral thesis

**Motivation:** The study of the space debris dynamics, the design of disposal orbits for MEO satellites and the need to describe accurately the dynamics around irregularly shaped bodies are just a few of the motivation behind our work.
**Methodology of Research:** Development of a model which describes space debris dynamics around extended bodies such as the Earth or an irregularly shaped body.
**Results and Comparison with State-of-the-art:** The obtained original results are consistent with the ones from other authors which use different techniques.
**Conclusions:** This work is the foundation for an upcoming paper on Solar Radiation Pressure resonances. The Polyhedron dynamics section is the result of the work of the Author with the Project Working Group "FLAIR" within the Stardust-r MCSA ITN.
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