ON SOME SUBCLASSES OF UNIVALENT FUNCTIONS ON THE UNIT DISC IN $\mathbb{C}$ AND ON THE EUCLIDEAN UNIT BALL IN $\mathbb{C}^n$

Eduard Ştefan Grigoriciuc[1]

[1] Doctoral School of the Faculty of Mathematics and Computer Science, "Babeş-Bolyai" University, Cluj-Napoca

eduardgrigoriciuc@yahoo.com

Abstract

In this paper we discuss about some new subclasses of univalent functions on the unit disc in $\mathbb{C}$, respectively univalent mappings on the Euclidean unit ball in $\mathbb{C}^n$, namely E_p and E_p^*, where $p \in \mathbb{N}$. In the first part, we present the particular class E_1 and some general results regarding to this class (duality theorems, estimations of coefficients, growth and distortion result, a general distortion theorem - some upper bounds for the modulus of the k-th derivative of a function, $k \in \mathbb{N}$ - and some examples). We also present the class E_p and the connection with the class K of convex functions. In this first part, we also construct some special Loewner chains using the functions from the class E_p. The second part contains results regarding to the classes $E_p(\mathbb{B}^n)$ and $E_p^*(\mathbb{B}^n)$ on the Euclidean unit ball in $\mathbb{C}^n$. For this subclasses we present also some examples on $\mathbb{B}^n$ obtained by using extension operators (for example, the Roper-Suffridge operator and generalization of this operator given by I. Graham, G. Kohr and M. Kohr).

Keywords: univalent functions; convexity; starlikeness; convexity of order alpha; Loewner chains
Domain: mathematics
Section: New (2020) thesis proposals

Motivation

The theory of univalent functions is an interesting topic both in the case of functions of a complex variable, but especially in the n-dimensional case where both the results and the proof methods take different forms. Among those who contributed to the development of this domain we mention A.W. Goodman, C. Pommerenke, P.L. Duren, T.J. Suffrdige, J.A. Pfaltzgraff, I. Graham, G. Kohr, M. Kohr, H. Hamada, P.T. Mocanu and other whose results are the basis of this work. In this paper we want to construct new subclasses of univalent functions on the unit disc in $\mathbb{C}$ starting from the well-know class K of convex functions. Also, we want to extend these subclasses in the n-dimensional case.

Methodology of Research

We study the new subclasses E_p and E_p^* using the duality results between K and E_p, respectively S^* and E_p^*. In the case $n = 1$ we use the classical methods developed by the mathematicians mentioned above. For the n-dimensional case, we describe the subclasses $E_p(\mathbb{B}^n)$ and $E_p^*(\mathbb{B}^n)$ on the Euclidean unit ball $\mathbb{B}^n$ using especially the results of Suffridge, Pfaltzgraff, Graham, Kohr, Gong. In the elaboration of this papee we use both classic articles and also recent articles by Graham, Kohr, Hamada and Bracci.

Results and Comparison with State-of-the-art

We obtain some general results for the subclasses E_p and E_p^* such as growth and distortion results, estimates of coefficients, duality theorems, structure formulas. Some of the results can be seen as particular cases of results for convex functions, but can also be treated strictly in the context of these new subclasses.

Conclusions

Finally, we present some further research directions regarding to these new subclasses of functions: connection of the classes with the theory of Loewner chains, study of compactness of the classes, study of extreme and support points, provide examples of univalent mappings on the unit ball in $\mathbb{C}^n$ using classical extension operators or new extension operators which preserve the geometric properties from one to several complex variables.

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