CUBIC MICROLATTICES EMBEDDED IN NEMATIC LIQUID CRYSTALS: A LANDAU-DE GENNES STUDY

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Abstract

We consider a Landau-de Gennes model for a connected cubic lattice scaffold in a nematic host, in a dilute regime. We analyse the homogenised limit for both cases in which the lattice of embedded particles presents or not cubic symmetry and then we compute the free effective energy of the composite material.

In the cubic symmetry case, we impose different types of surface anchoring energy densities, such as quartic, Rapini-Papoular or more general versions, and, in this case, we show that we can tune any coefficient from the corresponding bulk potential, especially the phase transition temperature.

In the case with loss of cubic symmetry, we prove similar results in which the effective free energy functional has now an additional term, which describes a change in the preferred alignment of the liquid crystal particles inside the domain.

Moreover, we compute the rate of convergence for how fast the surface energies converge to the homogenised one and also for how fast the minimisers of the free energies tend to the minimiser of the homogenised free energy.

Keywords: Nematic liquid crystals, Homogenisation problem, Gamma-convergence

Domain: Mathematics

Section: Elaboration of the doctoral thesis

Motivation: Liquid crystal materials are used in a wide variety of technologies, one of the main examples being liquid crystal displays (LCD). Nematic liquid crystals can enter the liquid crystal state of matter depending on the temperature at which they are. For example, *N-(4-Methoxybenzylidene)-4-butylaniline* (MBBA) is a nematic liquid crystal which can enter this state of matter in a range of temperatures between 21°C and 46°C. While constructing LCD's, liquid crystal materials are often exposed to temperatures that are outside of this range, hence the desire to create mixtures with new properties.

Methodology of research: We construct this work based on the techniques developed in [1] and [2], but now focused more on the structure of the scaffold. Since in those two studies the embeddings are unions of disconnected particles, the difficulty arises when we use only one single connected 3D-object. We use tools such as Gamma-convergence, the Landau-de Gennes Q-tensor theory for nematic liquid crystals and the general principles for homogenisation problems presented in [3].

Results and Comparison with State-of-the-art: We prove that, using cubic microlattices embedded in a nematic liquid crystal in a dilute regime, the new material that is created can be treated as a new nematic liquid crystal, but now with different properties, the main difference being the modification of the range of temperatures at which the material can enter the liquid crystal state of matter. Moreover, we also prove a rate of convergence for how fast the surface energy functionals converge to the homogenised limit, which gives us a connection between the scaling of the scaffold used and how good the mixture is approximating a new nematic liquid crystal material.

Conclusions: The mathematical study of this homogenisation problem approves what has been already observed in industry: we can design a scaffold such that the mixture between nematic liquid crystal molecules and the cubic microlattice creates a new material that behaves like a nematic liquid crystal, but now with the desired properties.

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