ROPER-SUFFRIDGE EXTENSION OPERATORS AND JANOWSKI UNIVALENT FUNCTIONS

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Abstract. In this paper, we will present certain properties that are satisfied on the unit ball $\mathbf{B}^n$ by the following Roper-Suffridge extension operators:

$$\Phi_{n,\alpha,\beta}(f)(z) = \left(f(z_1), \tilde{z}\left(\frac{f(z_1)}{z_1}\right)^\alpha (f'(z_1))^\beta \right), \ z = (z_1, \tilde{z}) \in \mathbf{B}^n,$$

where $\alpha, \beta \geq 0$, and

$$\Phi_{n,Q}(f)(z) = (f(z_1) + f'(z_1)Q(\tilde{z}), \tilde{z}\sqrt{f'(z_1)}), \ z = (z_1, \tilde{z}) \in \mathbf{B}^n,$$

where $Q : \mathbb{C}^{n-1} \to C$ is a homogeneous polynomial of degree 2. We will show that the above mentioned extension operators preserve the g-parametric representation, where the function g is given by $g(\zeta) = \frac{1+A\zeta}{1+B\zeta}$, $\zeta \in U$ and $-1 \leq B < A \leq 1$. Also, these extension operators preserve the Janowski starlikeness and the Janowski almost starlikeness. Other particular cases will also be mentioned.

Keywords: g-Loewner chain; g-parametric representation; Janowski starlikeness; Janowski almost starlikeness

Domain: Mathematics
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Our main purpose is to show that certain Roper-Suffridge extension operators preserve the notion of g-parametric representation. In [1,2], we studied the extension operators $\Phi_{n,\alpha,\beta}(f)(z) = \left(f(z_1), \tilde{z}\left(\frac{f(z_1)}{z_1}\right)^\alpha (f'(z_1))^\beta \right)$, $z = (z_1, \tilde{z}) \in \mathbf{B}^n$, $\alpha, \beta \geq 0$ (see [3]) and $\Phi_{n,Q}(f)(z) = (f(z_1) + f'(z_1)Q(\tilde{z}), \tilde{z}\sqrt{f'(z_1)})$, $z = (z_1, \tilde{z}) \in \mathbf{B}^n$, where $Q : \mathbb{C}^{n-1} \to C$ is a homogeneous polynomial of degree 2 (see [4]).

In order to prove this, we used the notion of g-Loewner chain ([5]) and well-known results in the theory of holomorphic mappings on the unit ball $\mathbf{B}^n$ and, also, in the theory of univalent functions on the unit disk U.

The main results highlight that the extension operators $\Phi_{n,\alpha,\beta}$, respectively $\Phi_{n,Q}$, preserve the g-parametric representation on unit ball $\mathbf{B}^n$ for $\alpha \in [0,1]$, $\beta \in [0,1/2]$ and $\alpha + \beta \leq 1$, respectively for $\|Q\| \leq \frac{A-B}{4(1+|B|)}$. Next, we extended our study to some subclasses of holomorphic mappings that have g-parametric representation

and obtained that the extension operators preserve the Janowski starlikeness and the Janowski almost starlikeness.

Well-known results are obtained as particular cases.

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