GALOIS CONNECTIONS IN MODULE THEORY

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A Galois connection is a pair (f,g) of order-preserving mappings $f\colon P \longleftrightarrow Q :g$ between ordered structures $(P, \leq)$ and $(Q, \leq)$. Such pairs are characterized by the property of adjointness, $f(p) \leq q \Longleftrightarrow p \leq g(q)$, for every $p \in P$ and $q \in Q$. Thus, we have a pair consisting of a left and right adjoints. Two examples of Galois connections are (i) isomorphisms of order, (f, f^{-1}), and, (ii) if R is a commutative ring with unity, then multiplication with a fixed ideal of R is a left adjoint with respect to the inclusion-ordered set of ideals of R [7]. The examples make clear that we can learn more about properties of ordered structures, and modules in particular, by studying Galois connections between them. A first step is to understand the meaning of Galois connections between $\mathbb{Z}$-modules, ie. between additive abelian groups. We report in this contribution some results obtained during the first year of studies concerning the enumeration of Galois connections between the subgroup lattice $L(\mathbb{Z}(p^k \times p^l), \leq)$ to itself. The next step is to study the relation between properties a module and its endomorphism ring, as has been done before in the literature [3,4].

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