Polynomial dichotomy in Banach spaces

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Abstract

The qualitative theory of asymptotic behavior for dynamical systems in Banach spaces such as stability, instability, dichotomy and trichotomy have registered a notable growth in the last years. The concept of exponential dichotomy has been studied for many years. Even though nowadays it plays an important role in the theory of dynamical systems, there are some situations in which the notion of exponential dichotomy is too restrictive for the dynamics and for this reason it is important to have in mind a more general type of dichotomic behavior. In this sense, we refer to the polynomial dichotomy notion, which was firstly mentioned in 2009.

The main objective of this paper is to describe the polynomial dichotomy behavior. We give characterization theorems for both uniform and nonuniform polynomial dichotomy concept for evolution operators in Banach spaces. Moreover, some necessary and sufficient conditions for the nonuniform polynomial dichotomy concept with respect to a family of norms are presented.

Keywords: evolution operators, polynomial dichotomy, family of norms

Domain: mathematics

Section: Elaboration of the doctoral thesis

Motivation: The notion of exponential dichotomy is too stringent for the dynamics and it is of considerable interest to look for more general types of dichotomic behaviors. The main reason is that from the point of view of ergodic theory almost all variational equations in a finite-dimensional space admit a nonuniform type of behavior.

Methodology of Research: Our approach is based on the extension of techniques for exponential dichotomy to the case of polynomial dichotomy.

Results and Comparison with State-of-the-art: We give some necessary and sufficient conditions for the nonuniform polynomial dichotomy concept with respect to a family of norms, an idea which was introduced by L. Barreira, D. Dragičević and C. Valls [1] in 2015 for the exponential case.

Conclusions: We obtain characterization theorems that are generalizations of well known results in the field of asymptotic behavior of evolution operators, theorems that can have a significant contribution in the development of this research area.

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