ON GRACEFUL AND HARMONIOUS COLORINGS OF PARTICULAR GRAPHS

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Abstract

Keywords: coloring; graceful chromatic number; harmonious chromatic number
Domain: computer science
Section: Elaboration of the doctoral thesis

Motivation. Graph colorings are a fundamental topic in graph theory and originate from the Four Color Problem of Francis Guthrie from 1852. Since then, researchers have been studying the subject intensively. The most known type of coloring is the proper vertex k-coloring in which the goal is to color the vertices of an undirected graph with k colors such that any two adjacent vertices are colored with distinct colors. Our central focus is two types of coloring, the graceful coloring, introduced by Gary Chartrand in 2015, and the harmonious coloring, proposed in 1982 by Harary and Plantholt.

Methodology of Research. Any graph has a graceful coloring, and a harmonious coloring, however for graph G is not easy to find the graceful chromatic number and the harmonious chromatic number. Determining the harmonious chromatic number is an NP-complete problem. There are not general algorithms that can compute these two numbers for any graph. We can only study particular graphs or classes of graphs. We wrote recursive algorithms to validate the theoretical results and to obtain different examples of graceful coloring and harmonious coloring with the same number of colors for the same graph, respectively.

Results and Comparison with State-of-the-art. We find the graceful chromatic number for some well-known graphs, both individual and classes of graphs, such as the diamond graph, Petersen graph, Moser spindle graph, Goldner-Harary graph, friendship graphs, fan graphs, Dürer graph, Heawood graph, Desarques graph, Möbius-Kantor graph, Nauru graph, Tietze's graph, Golomb graph, cactus, Gear, web graphs, etc. We also studied the harmonious chromatic number for many of these graphs.

Conclusions. We find the graceful chromatic and the harmonious chromatic number for several particular graphs and classes of graphs. We noticed that, for a graph G, either the graceful chromatic number and the harmonious chromatic number are identical or differ very little.

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