ON TRANSLATED RANK-2 BRILL-NOETHER LOCI ON HIRZEBRUCH SURFACES

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Abstract

The classical Brill-Noether theory has proved to be a prolific source of geometrical information in the study of the moduli spaces $Pic^d(C)$ of degree d line bundles on a smooth projective curve C of genus g. Given the major impact of the theory of Alexander von Brill and Max Noether, the numerous attempts at generalizing it are natural. One can look at stable rank r vector bundles on a curve C having at least k independent sections or study line bundles with at least k independent sections on a variety of arbitrary dimension or, more generally, go both ways. In their paper [5], L. Costa and R. M. Miró-Roig considered the sets $W_H^k(r; c_1, \ldots, c_s)$ of H-stable rank r vector bundles on a smooth projective variety n-dimensional X which have at least k independent sections. They proved that these $W_H^k(r; c_1, \ldots, c_s)$ are determinantal varieties provided that certain cohomology groups vanish for all vector bundles $E \in M_H(r; c_1, \ldots, c_s)$. Moreover, they have showed that on a Hirzebruch surface, for a suitable choice of Chern classes, the Brill-Noether loci $W_H^1(2; c_1, c_2)$ describe the difference between moduli spaces of vector bundles on X, stable with respect to different polarizations.

The aim of this work is to extend the results of Costa and Miró-Roig in [5], building upon the foundational work of Qin ([8],[9] and [10]). Specifically, we define the translated Brill-Noether loci as being twists of Brill-Noether loci with a fixed line bundle and we investigate their connections with the extension spaces defined by Qin. Furthermore, we investigate their non-emptyness via the moduli space $M(c_1, c_2, d, r)$ constructed by Brînzănescu and Stoia (for details see [1], [2] and [3]). Eventually, we show that the dimension of the translated Brill-Noether loci is equal to the expected dimension.

Keywords: translated Brill-Noether loci; Hirzebruch surfaces; rank-2 bundles

Domain: Mathematics

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