# GROUP GRADED MORITA EQUIVALENCES OVER A GROUP GRADED GROUP ACTED ALGEBRA. PROPERTIES

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## Abstract

**Keywords:** Group graded algebras; wreath products; Morita equivalences; crossed products; centralizer subalgebra; character triples
**Domain:** Mathematics
**Section:** Elaboration of the doctoral thesis

## Motivation

An important strategy used in the verification of the local-global conjectures of the representation theory of finite groups is to reduce the statements to some stronger ones about simple groups. In order to obtain such reduction theorems the language of character triples and of the relations between them has proven useful, as surveyed by Britta Späth in [4, 5].

As per the conviction that character correspondences with good properties are consequences of categorical equivalences, we have proved that there is a link between character triples and group graded Morita equivalences with some additional properties [1].

## Methodology of Research

In order to obtain the aforementioned link, we have used the method of graded algebras. To explain this link, let us introduce our context.

Let $G$ be a finite group and $N$ a normal subgroup of $G$. We denote by $\bar{G} := G/N$. We consider $\mathscr{C}$ to be a $\bar{G}$-graded $\bar{G}$-acted algebra. Assume that we have two $\bar{G}$-graded $\bar{G}$-acted algebra homomorphisms from $\mathscr{C}$ to the centralizer of $B$ in $A$, and of $B'$ in $A'$ respectively, denoted by $\zeta$ and $\zeta'$.

The relation $\leq_c$ between character triples ([5, Definition 2.7]) leads us to the consideration of $\bar{G}$-graded $(A,A')$-bimodules $\tilde{M}$ satisfying $m_{\bar{g}}\zeta'(c) = \zeta({}^{\bar{g}}c)m_{\bar{g}}$ for all $\bar{g} \in \bar{G}$, $c \in \mathscr{C}$ and $m_{\bar{g}} \in \tilde{M}_{\bar{g}}$.

## Results

We obtain that equivalences induced by such bimodules imply the relation $\leq_c$ between corresponding character triples ([1, Theorem 6.7]). This gives the motivation behind the study of such Morita equivalences, which we call $\bar{G}$-graded $\bar{G}$-acted Morita equivalences over $\mathscr{C}$.

Furthermore, we will present our development of the group graded Morita theory over $\mathscr{C}$, as we have done in [2].

Finally, we will present some of the properties that such equivalences have, as we have shown in [3], i.e. group graded Morita equivalences over $\mathscr{C}$ are compatible in a certain sense with tensor and wreath products.

## Conclusions

We have developed the notion of group graded Morita equivalences over a group graded group acted algebra, which can be utilized to obtain relations between character triples, relations which are useful in obtaining reduction theorems for the local-global conjectures.

## References


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