

# On a special class of fractional-type set-valued functions

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## Abstract

The ratios of affine functions have been introduced and studied by Uriel G. Rothblum in 1985 in the framework of finite dimensional real spaces. These functions have the property of preserving the convexity of sets through direct and inverse images. Within a recent paper, Alexandru Orzan has proposed a similar concept for set-valued maps and has shown that these functions also turn convex sets into convex sets. The aim of this paper is to characterize this new class of set-valued functions and, at the same time, to investigate the convexity preserving property for sets as far as the inverse upper and lower images are concerned.

**Keywords:** Generalized Convexity, Set-valued Functions, Fractional-type functions.

**Domain:** Mathematics.

**Section:** New (2020) thesis proposal.

## Motivation

The concept of fractional-type functions play an important role in many fields such as mathematics, physics, economy, biology and so on.

## Methodology of Research

Throughout this endeavor we will make use of a variety of theorems and results from convex analysis and optimization. Besides that, we shall be using many ideas from already well established proofs within the papers pointed out in the bibliography

## Results and Comparison with State-of-the-art

The main results of this work are the characterization theorem concerning our class of set-valued functions and the convexity preserving properties for convex sets through inverse images. By using these results, one can easily find the original results stated by Rothblum for vector-valued functions.

## Conclusions

Whilst vector-valued ratios of affine functions have certain interesting properties concerning the preservation of convexity, we have shown that the class of ratios of affine set-valued functions (in the sense of Tan) share the same property.

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