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Information Processing Letters 96 (2005) 54–58

Information Processing Letters

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# Structural soundness of workflow nets is decidable [☆]

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Received 7 January 2005; received in revised form 16 May 2005; accepted 4 June 2005

Available online 28 June 2005

Communicated by L.A. Hemaspaandra

**Abstract**

The problem of deciding whether a given workflow net is $k$-sound for some $k \geqslant 1$ is known as *structural soundness*. We prove that structural soundness of workflow nets is decidable.

*Keywords:* Petri net; Workflow net; Soundness; Decidability; Algorithms

## 1. Introduction and preliminaries

Petri nets have a well-defined semantics, they provide a graphical language, are expressive, and many properties and analysis techniques for Petri nets are available. Thus, it is not surprising that for some time Petri nets have been used for *workflow modeling*, see [2] for a good introduction to the subject.

*Proper termination*, also called *soundness* is a correctness property of a workflow net. This property is formulated with respect to an initial and a final marking which consists of a single token on the initial and, respectively, final place. The soundness property is equivalent to liveness and boundedness, which are decidable properties, thus it can be verified by standard Petri net methods [1].

A natural extension of soundness, called $k$-soundness [5], allows $k$ tokens on the initial and final place, where $k \geqslant 1$. This extension, which was proved decidable in [5], gives rise to another two related soundness concepts: *generalized soundness* [7], which means $k$-soundness for all $k \geqslant 1$, and *structural soundness* [5], which means $k$-soundness for some $k \geqslant 1$.

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[☆] The research reported in this paper was partially supported by National Science Foundation, Grants MCB9527131, DBI0296107, ACI0296035, and EIA0296179, and by National University Research Council of Romania, Grants CNCSIS 28/632/2004 and CNCSIS 50/632/2005.

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doi:10.1016/j.ipl.2005.06.002

Generalized soundness is decidable [8] and in this paper we will prove that structural soundness is decidable too.

Our study on structural soundness has multiple motivations. To begin with, we mention that generalized soundness is an important property of workflow nets but, unfortunately, hard to establish because it requires $k$-soundness for all $k$. In practice, often it is sufficient to know that a workflow net is $k$-sound for some $k$ and for a set of values related to $k$ (i.e., it is structurally sound).

Several techniques for enhancing the performance of systems by allowing iteratively more cases to enter the system and still preserve structural soundness property have been proposed in [5] and in [4], and in [3]. The main idea is to add "local controls" at each step and then to check the structural soundness of the new net obtained in this way. In some cases, such an iterative process eventually stops [4] (i.e., a step is reached where the local control added can be played by an already existing control). The main problem now is to decide structural soundness of a workflow net obtained from an existing one by some transformations.

For circuit-free workflow nets, structural soundness can be characterized by a kind of "structural boundedness together with quasi-liveness and controlled-siphon property" [5]. However, this characterization does not lead to a decision procedure for structural soundness, unless a decision procedure for "structural boundedness together with quasi-liveness and controlled-siphon property" is developed.

There is another reason which motivates our work on structural soundness. As shown in the paper, if a workflow net is $k$-sound for some $k > 1$, and $k$ is the smallest integer with this property, then the workflow net is not $k'$-sound, for any $k'$ which is not a multiple of $k$. Therefore, this result tells us what cases in the above iterative process can be avoided and, as a result, checking structural soundness of the original workflow net and determining $k$ would reduce the problem to $k'$-soundness tests, for some $k'$ multiple of $k$.

Finally, the study of structural soundness is strongly advocated by the necessity of a clear and complete understanding of parameterized soundness. In addition to the decidability of structural soundness, our note shows that a workflow net that is not generalized sound can be $k'$-sound only for multiples of some value $k$ that can be effectively computed. Therefore, once such a value $k$ is known, we implicitly know all values $k'$ for which the workflow net is not $k'$-sound. We believe that all these make a further step in understanding parameterized soundness.

The rest of this section reviews some basic concepts and notations for Petri nets and workflow nets. Section 2 contains the proofs of our main result.

A *Petri net* [9] is a tuple $\Sigma = (S, T, F, W)$, where $S$ and $T$ are two finite sets (of *places* and *transitions*, respectively), $S \cap T = \emptyset$, $F \subseteq (S \times T) \cup (T \times S)$ is the *flow relation*, and $W : (S \times T) \cup (T \times S) \to \mathbf{N}$ is the *weight function* of $\Sigma$ verifying $W(x, y) = 0$ iff $(x, y) \notin F$ ($\mathbf{N}$ being the set of natural numbers). Given $x \in S \cup T$ we denote ${}^\bullet x = \{y \mid (y, x) \in F\}$ and $x^\bullet = \{y \mid (x, y) \in F\}$.

A *marking* of $\Sigma$ is any function $M \in \mathbf{N}^S$ from $S$ into $\mathbf{N}$, usually denoted as an $S$-indexed vector. The *transition relation* of a Petri net $\Sigma$ states that a transition $t$ is *enabled* at a marking $M$, denoted by $M[t\rangle_\Sigma$, if $M(s) \geqslant W(s, t)$ for all $s \in S$. If $t$ is enabled at $M$, then it can fire yielding a new marking $M'$ given by $M'(s) = M(s) - W(s, t) + W(t, s)$ for all $s \in S$; we denote this by $M[t\rangle_\Sigma M'$. The transition relation is extended usually to sequences of transitions. When there is a sequence $w \in T^*$ such that $M[w\rangle_\Sigma M'$ we say that $M'$ is *reachable* (from $M$ in $\Sigma$). We denote by $[M\rangle_\Sigma$ the set of all reachable markings (from $M$) in $\Sigma$. When no confusion may arise we simplify the notation $[\cdot\rangle_\Sigma$ to $[\cdot\rangle$.

A *workflow net* [1] (WF net) is a Petri net $\Sigma$ with the following two properties:

(1) $\Sigma$ has two special places $i$ and $o$ called the *input* and respectively the *output place* of $\Sigma$. They satisfy ${}^\bullet i = \emptyset$ and $o^\bullet = \emptyset$;
(2) Any node $x \in S \cup T$ in the graph of $\Sigma$ is on a path from $i$ to $o$.

Given a WF net $\Sigma$, a place $s$ of it, and a natural number $k \geqslant 1$, we denote by $M_{ks}$ the marking given by $M_{ks}(s) = k$ and $M_{ks}(s') = 0$ for all $s' \neq s$. When $k = 1$ the notation is simplified to $M_s$.

$k$-soundness has been introduced in [5], but we will follow the definition in [7,8]. The difference is that the definition in [7,8] does not require the absence of dead transitions, focusing only on the proper termination property which is the crucial one. Therefore, according to [7,8], a WF net $\Sigma$ is called *$k$-sound*, where $k \geqslant 1$, if every reachable marking $M \in [M_{ki}\rangle$ terminates properly, i.e., $M_{ko} \in [M\rangle$. $\Sigma$ is *structurally sound* if it is $k$-sound for some $k \geqslant 1$.

## 2. Deciding structural soundness

We prove in this section that the structural soundness property for WF nets is decidable.

**Definition 2.1.** Let $\Sigma = (S, T, F, W)$ be a WF net and $k \geqslant 1$.

(1) A *$k$-initiated computation* or a *$k$-computation* of $\Sigma$ is any computation of the form $M_{ki}[u\rangle M$, where $u \in T^*$ and $M \in N^S$. Recall that $i$ is the input place and the marking $M_{ki}$ has the following properties: $M_{ki}(i) = k$ and $M_{ki}(s') = 0$ for all $s' \neq i$.

(2) A $k$-computation $M_{ki}[u\rangle M$ is called *sound* if there exists $v \in T^*$ such that $M[v\rangle M_{ko}$.

Now, it is clear that a WF net $\Sigma$ is $k$-sound if and only if all its $k$-computations are sound. Moreover, we say that $\Sigma$ is *quasi $k$-sound* if $\Sigma$ has at least one sound $k$-computation (i.e., $M_{ko} \in [M_{ki}\rangle_\Sigma$). If $\Sigma$ is quasi $k$-sound for some $k \geqslant 1$, then we will say that it is *structurally quasi-sound*. The *structural quasi-soundness problem* for WF nets is to decide whether a given WF net is structurally quasi-sound.

**Lemma 2.1.** *The structural quasi-soundness problem for WF nets is decidable.*

**Proof.** Let $\Sigma$ be a WF net. Construct the Petri net $\Sigma' = (S', T', F', W')$ by adding to $\Sigma$ places $p_1, p_2, p_3$ and transitions $r_1, r_2, r_3$, as shown in Fig. 1. Denote the markings of $\Sigma'$ as vectors $(M, x, y, z)$, where $M$ is a marking of $\Sigma$, $x$, $y$, and $z$ are the number of tokens in $p_1$, $p_2$, and $p_3$, respectively. The initial marking of $\Sigma'$ is $(\underline{0}, 1, 0, 0)$, where $\underline{0}$ denotes a vector whose components are all 0.

We prove that $\Sigma$ is structurally quasi-sound if and only if $(\underline{0}, 0, 0, 1)$ is reachable in $\Sigma'$.

If there exists $k \geqslant 1$ and $w \in T^*$ such that $M_{ki}[w\rangle_\Sigma M_{ko}$, then

$$(\underline{0}, 1, 0, 0)\big[r_1^k\big\rangle_{\Sigma'}(M_{ki}, 1, k, 0)[w\rangle_{\Sigma'} (M_{ko}, 1, k, 0)\big[r_2 r_3^k\big\rangle_{\Sigma'}(\underline{0}, 0, 0, 1)$$

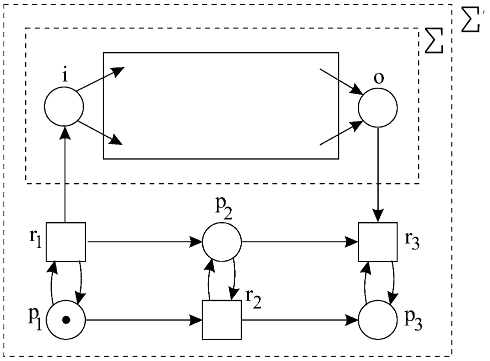

Fig. 1. The Petri net $\Sigma'$.

is a valid computation in $\Sigma'$ which shows that $(\underline{0}, 0, 0, 1)$ is reachable.

Conversely, assume that $(\underline{0}, 0, 0, 1)$ is reachable in $\Sigma'$. Then, there exists $w \in T'^*$ such that $(\underline{0}, 1, 0, 0)[w\rangle_{\Sigma'}(\underline{0}, 0, 0, 1)$. $w$ should contain at least one occurrence of $r_2$ and, therefore, at least one occurrence of $r_1$. Moreover, $r_2$ occurs exactly once in $w$. Let $w = ur_2v$ and $k$ be the number of occurrence of $r_1$ in $u$. It is easy to see that $r_1$ does not occur in $v$, $r_3$ does not occur in $u$, and the number of occurrence of $r_3$ in $v$ is exactly $k$. Let

$$(\underline{0}, 1, 0, 0)[u\rangle_{\Sigma'}(M, 1, k, 0)[r_2\rangle_{\Sigma'}(M, 0, k, 1)[v\rangle_{\Sigma'} (\underline{0}, 0, 0, 1),$$

for some $M$. Let $u'$ be the sequence obtained from $u$ by removing all $r_1$'s occurrences, and $v'$ obtained from $v$ by removing all $r_3$'s occurrences. As the place $i$ has no input arc in $\Sigma$ and the place $o$ has no output arc in $\Sigma$, the following is a valid computation in $\Sigma'$

$$(\underline{0}, 1, 0, 0)\big[r_1^k u'\big\rangle_{\Sigma'}(M, 1, k, 0)[r_2\rangle_{\Sigma'} (M, 0, k, 1)\big[v' r_3^k\big\rangle_{\Sigma'}(\underline{0}, 0, 0, 1)$$

(i.e., all $r_1$'s occurrences can be applied at the very beginning and all $r_3$'s occurrences can be applied at the very end).

Now, it is straightforward to see that the computation

$$(\underline{0}, 1, 0, 0)\big[r_1^k u'\big\rangle_{\Sigma'}(M, 1, k, 0)$$

induces $M_{ki}[u'\rangle_\Sigma M$, and the computation

$$(M, 0, k, 1)\big[v' r_3^k\big\rangle_{\Sigma'}(\underline{0}, 0, 0, 1)$$

induces $M[v'\rangle_\Sigma M_{ko}$. That is, $M_{ki}[u'v'\rangle_\Sigma M_{ko}$, which shows that $\Sigma$ is structurally quasi-sound.

We conclude the proof by recalling that the reachability problem for Petri nets is decidable [6] and, therefore, the structural quasi-soundness problem is decidable. □

The Algorithm $\mathcal{A}_1$ decides whether a WF net is structurally quasi-sound.

**Algorithm $\mathcal{A}_1$.**

```
input:   WF net $\Sigma$;
output:  “Yes” if $\Sigma$ is structurally quasi-sound, and
         “No”, otherwise;
begin
    Construct $\Sigma'$ as in the proof of Lemma 2.1;
    if $(\underline{0}, 0, 0, 1)$ is reachable in $\Sigma'$ then “Yes”
    else “No”
end.
```

The Algorithm $\mathcal{A}_1$ has the same upper bound complexity as the reachability problem for Petri nets.

If a WF net $\Sigma$ is structurally quasi-sound then we are able to find the least $k$ such that $\Sigma$ is quasi $k$-sound. Let $k_\Sigma$ be this $k$. That is,

$$k_\Sigma = \min\{k \geqslant 1 \mid M_{ko} \in [M_{ki}\rangle_\Sigma\}.$$

**Lemma 2.2.** *Let $\Sigma$ be a structural quasi-sound WF net. If $k_\Sigma > 1$, then $\Sigma$ is not $k$-sound, for any $k \in \{mk_\Sigma + j \mid m \geqslant 1,\ 1 \leqslant j < k_\Sigma\}$.*

**Proof.** Assume that $\Sigma = (S, T, F, W)$ is a structural quasi-sound WF net, $k_\Sigma > 1$, and let $w \in T^*$ such that $M_{k_\Sigma i}[w\rangle M_{k_\Sigma o}$. Let $m \geqslant 1$ and $j$ such that $1 \leqslant j < k_\Sigma$. The following is a valid computation in $\Sigma$:

$$M_{(mk_\Sigma + j)i}[w^m\rangle M,$$

where $M(i) = j$, $M(o) = mk_\Sigma$ and $M(s) = 0$, for all $s \in S - \{i, o\}$.

As $1 \leqslant j < k_\Sigma$, $\Sigma$ is not quasi $j$-sound. Therefore, no transition sequence can lead $\Sigma$ from $M$ to $M_{(mk_\Sigma + j)o}$ because, otherwise, $\Sigma$ would have sound $j$-computations (the tokens in the place $o$ do not matter because this place has no output arc). This shows that the $(mk_\Sigma + j)$-computation above is not sound and, therefore, $\Sigma$ is not $(mk_\Sigma + j)$-sound. □

**Lemma 2.3.** *Let $\Sigma$ be a structural quasi-sound WF net. If $\Sigma$ is not $k_\Sigma$-sound, then $\Sigma$ is not $mk_\Sigma$-sound, for any $m \geqslant 1$.*

**Proof.** Assume that $\Sigma = (S, T, F, W)$ is structurally quasi-sound but not $k_\Sigma$-sound. Therefore, there exists a transition sequence $w \in T^*$ such that $M_{k_\Sigma i}[w\rangle M_{k_\Sigma o}$, and there exists a $k_\Sigma$-computation $M_{k_\Sigma i}[u\rangle M$ which is not sound.

Then, for any $m > 1$, the following is a valid computation in $\Sigma$:

$$M_{mk_\Sigma i}[w^{m-1}u\rangle M',$$

where $M'(o) = M(o) + (m - 1)k_\Sigma$ and $M'(s) = M(s)$, for all $s \in S - \{o\}$. No transition sequence can lead $M'$ to $M_{mk_\Sigma o}$ because $M_{k_\Sigma i}[u\rangle M$ is not a sound $k_\Sigma$-computation (the tokens in the place $o$ do not matter because this place has no output arc). This shows that the $mk_\Sigma$-computation above is not sound and, therefore, $\Sigma$ is not $mk_\Sigma$-sound. □

**Corollary 2.1.** *Let $\Sigma$ be a structural sound WF net and let $k$ be the least positive integer such that $\Sigma$ is $k$-sound. Then, $k = k_\Sigma$.*

**Proof.** If $k = 1$ then $k_\Sigma = 1$ too. Assume that $k > 1$. Then, $k_\Sigma \leqslant k$. If $k_\Sigma < k$, then two cases are to be considered:

(1) $k$ is a multiple of $k_\Sigma$. As $\Sigma$ is not $k_\Sigma$-sound but it is $k$-sound, this case contradicts Lemma 2.3;
(2) $k$ is not a multiple of $k_\Sigma$. Then, this case contradicts Lemma 2.2 because $\Sigma$ is not $k_\Sigma$-sound but it is $k$-sound.

Both cases lead to a contradiction and, therefore, we conclude that $k = k_\Sigma$. □

**Theorem 2.1.** *Let $\Sigma$ be a WF net. Then, $\Sigma$ is structurally sound if and only if $\Sigma$ is structurally quasi-sound and $k_\Sigma$-sound.*

**Proof.** From Lemma 2.1 and Corollary 2.1. □

The Algorithm $\mathcal{A}_2$ decides whether a WF net is structurally sound.

**Algorithm $\mathcal{A}_2$.**

```
input:  WF net $\Sigma$;
output: “Yes” if $\Sigma$ is structurally sound, and “No”,
           otherwise;
begin
    if $\Sigma$ is not structurally quasi-sound then “No”
        else begin
                compute $k_\Sigma$;
                if $\Sigma$ is $k_\Sigma$-sound then “Yes”
                else “No”
             end
end.
```

The correctness of this algorithm follows easily from Theorem 2.1 and from the fact that $k$-soundness is decidable for WF nets [5]. The complexity of this algorithm is dominated by the complexity of the reachability problem [6] which is used to solve the structural quasisoundness problem and by the problem of finding $k_\Sigma$ (in case that $\Sigma$ is structurally quasi-sound). We believe that the reachability algorithm for Petri nets can be designed in such a way that it returns $k_\Sigma$ as well, when the input WF net $\Sigma$ is bounded, but we do not yet have a solution to this problem.

We conclude our paper with an open problem.

**Open problem.** If a WF net $\Sigma$ is structurally sound, then it is $k_\Sigma$-sound. If $k_\Sigma > 1$, then we know that $\Sigma$ is not $k$-sound for any $k < k_\Sigma$ and any $k < mk_\Sigma + j$, where $m \geqslant 1$ and $1 \leqslant j < k_\Sigma$. Now, the problem is to investigate the $k$-soundness of $\Sigma$ for $k = mk_\Sigma$, $m \geqslant 2$.

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